

Bidirectional and tunable single-photons multi-channel quantum router between microwave and optical light

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Routing of photon play a key role in optical communication and quantum networks. Although the quantum routing of signals has been investigated in various systems both in theory and experiment. However, no current theory can route quantum signals between microwave and optical light. Here, we propose an experimentally accessible tunable multi-channel quantum routing proposal using photon-phonon translation in a hybrid opto-electromechanical system. It is the first demonstration that the single-photon of optical frequency can be routed into three different output ports by adjusting microwave power. More important, the two output signals can be selected according to microwave power. Meanwhile, we also demonstrate the vacuum and thermal noise will be insignificant for the optical performance of the single-photon router at temperature of the order of 20 mK. Our proposal may have paved a new avenue towards multi-channel router and quantum network.

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Quantum information science has been developed rapidly due to the substitution of photons as signal carriers rather than the limited electrons [1]. Single photons are suitable candidates as the carrier of quantum information due to the fact that they propagate fast and interact rarely with the environment. Meanwhile, a quantum single-photons router is challenging because the interaction between individual photons is generally very weak. Quantum router or quantum switch plays a key role in optical communication networks and quantum information processing. It is important for controlling the path of the quantum signal with fixed Internet Protocol (IP) addresses, or quantum switch without fixed IP addresses.

Designing a quantum router or an optical switch operated at a single photon level enables a selective quantum channel in quantum information and quantum networks [2–9], such as in different systems, cavity QED system [10], circuit QED system [3], optomechanical system [4], a pure linear optical system [11, 12], Λ -type three-level system [13–15]. The essence lying at the core is the realization of the strong coupling between the photons and photons or photons and phonons [17–20], but these methods require high-pump-laser powers due to the very weak optical nonlinearity. To the best of our knowledge, the quantum router demonstrated in most experiments and theoretical proposals has only one output terminal, except for only a few theoretical method in Ref. [13–16] and the experiments in Ref.[12]. However, until now, the

above all of routers are applied only in optical light or only in microwave separately.

On the other hand, with the technological advancements in the fields of optical nanocavity and microwave circuit, it is now possible to engineer interactions between optical and microwave using photon-phonon translation. Both microwave and optical light have been separately used to cool a nanomechanical-resonator (NR) to its quantum ground state of motion [21, 22] and work in the strong coupling regime [23, 24]. This same interaction enables the mechanical resonator to serve as an information storage medium[25, 26], and opens up the possibility of high-fidelity frequency conversion [27–30].

In this letter, by combining the technologies of optomechanics and electromechanics, we simultaneously couple a NR to both a microwave circuit and an optical toroidal nanocavity. We show it is the first demonstration that the multi-channel quantum router can be tunable between microwave and optical light in this hybrid system. More important, the two output signals frequencies can be selected according to the microwave power. The thermal noise could be more critical in deteriorating the performance of the single-photon router. Then, we also demonstrate the vacuum and thermal noise can be insignificant for the optical performance of the single-photon router at temperature of the order of 20 mK.

Model setup and solutions.— The model for realizing tunable and bidirectional multi-channel quantum router is sketched in Fig. 1, where a microwave circuit and an optical toroidal nanocavity are coupled to a common NR via capacitively coupling [24] and evanescent coupling [31], respectively. The optical toroidal nanocavity with resonance frequency ω_1 is driven by a strong pump beam ε_l with frequency ω_l and a probe beam in a single-

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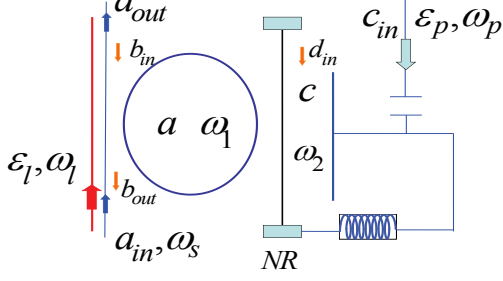


FIG. 1: (Color online) Schematic diagram of the hybrid system. Which consist of microwave circuit and an optical toroidal nanocavity are coupled to a common NR via capacitively coupling and evanescent coupling, respectively. The optical toroidal nanocavity with resonance frequency ω_1 is driven by a strong pump beam ε_l with frequency ω_l and a probe beam in a single-photon Fock state with frequency ω_s simultaneously. The microwave cavity with resonance frequency ω_1 is driven by a strong pump beam ε_p with frequency ω_p .

photon Fock state with frequency ω_s simultaneously. The microwave circuit with resonance frequency ω_2 is only driven by a strong pump beam ε_p with frequency ω_p . In the rotating frame at the frequency ω_l and ω_p , the Hamiltonian of the hybrid system can be written as

$$H = \hbar\Delta_a a^\dagger a + \hbar\Delta_c c^\dagger c + \frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2 - \hbar g_1 a^\dagger a q + \hbar g_2 c^\dagger c q + i\hbar\varepsilon_l(a^\dagger - a) + i\hbar\varepsilon_p(c^\dagger - c). \quad (1)$$

Here, a (a^\dagger) and c (c^\dagger) are the annihilation (creation) operator of optical toroidal nanocavity and microwave circuit, respectively. $\Delta_a = \omega_1 - \omega_l$, $\Delta_c = \omega_2 - \omega_p$ are the corresponding cavity-pump field detunings. p and q are the momentum and the position operator of the NR, respectively. The NR with frequency ω_m and effective mass m . g_1 (g_2) is the single-photon coupling rate between the mechanical mode and the optical (microwave) mode [32]. The last two terms in Eq.(1) describe the interaction between the cavity field (optical and microwave) with the input fields. The pump field strength ε_l (ε_p) depends on the power \wp_l (\wp_p) of coupling field, $\varepsilon_l = \sqrt{2\kappa_1\wp_l/\omega_l}$, ($\varepsilon_p = \sqrt{2\kappa_2\wp_p/\omega_p}$) with κ_1 (κ_2) is the optical toroidal nanocavity (microwave circuit) decay rate.

Note that the NR coupled to the thermal surrounding at the temperature T , which results in the mechanical damping rate γ_m , and thermal noise force ξ with frequency-domain correlation [4],

$$\langle \xi(\omega)\xi(\Omega) \rangle = 2\pi\hbar\gamma_m m\omega [1 + \coth(\frac{\hbar\omega}{2\kappa_B T})] \delta(\omega + \Omega), \quad (2)$$

where κ_B is the Boltzmann constant. In addition, the cavity field a is coupled to the input quantum fields a_{in}

and b_{in} . If there are no photons incident from the other direction, then b_{in} would be the vacuum field. Let $2\kappa_1$ be the decay rate at which photons leak out from the optical toroidal nanocavity. The output fields can be written as

$$x_{out}(\omega) = \sqrt{2\kappa_1}a(\omega) - x_{in}(\omega), \quad x = a, b. \quad (3)$$

These couplings are included in the standard way by writing quantum Langevin equations for the cavity field operators with the commutation relations $[a, a^\dagger] = 1$, $[c, c^\dagger] = 1$, $[p, q] = i\hbar$. Putting together all the quantum fields, thermal fluctuations, and the Heisenberg equations from the Hamiltonian (1), we can obtain the working quantum Langevin equations:

$$\begin{aligned} \dot{a} &= -[2\kappa_1 + i(\Delta_a + g_1 q)]a + \varepsilon_l + \sqrt{2\kappa_1}a_{in} + \sqrt{2\kappa_1}b_{in}, \\ \dot{c} &= -[2\kappa_2 + i(\Delta_c - g_2 q)]c + \varepsilon_p + \sqrt{2\kappa_2}c_{in} + \sqrt{2\kappa_2}d_{in}, \\ \dot{q} &= \frac{p}{m}, \\ \dot{p} &= -m\omega_m^2 q - \hbar g_1 a^\dagger a + \hbar g_2 c^\dagger c - \gamma_m p + \xi, \end{aligned} \quad (4)$$

The quantum Langevin equations (4) can be solved after all operator are linearized as its steady-state mean value and a small fluctuation:

$$p = p_s + \delta p, \quad q = q_s + \delta q, \quad a = a_s + \delta a, \quad c = c_s + \delta c, \quad (5)$$

where δp , δq , δa , δc being the small fluctuations around the corresponding steady values. After substituting Eq.(5) into Eq.(4), ignoring the second-order small terms, and introducing the Fourier transforms $f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\omega) e^{-i\omega t} d\omega$, $f^+(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f^*(-\omega) e^{-i\omega t} d\omega$. We can get the steady values

$$\begin{aligned} p_s &= 0, \quad q_s = \frac{\hbar g_2 |c_s|^2 - \hbar g_1 |a_s|^2}{m\omega_m^2}, \\ a_s &= \frac{\varepsilon_l}{2\kappa_1 + i\Delta_1}, \quad c_s = \frac{\varepsilon_p}{2\kappa_2 + i\Delta_2}, \end{aligned} \quad (6)$$

with $\Delta_1 = \Delta_a + g_1 q_s$, $\Delta_2 = \Delta_c - g_2 q_s$ and the solution of δa [33],

$$\begin{aligned} \delta a &= E_1(\omega)a_{in}(\omega) + F_1(\omega)a_{in}^+(-\omega) + E_1(\omega)b_{in}(\omega) \\ &+ F_1(\omega)b_{in}^+(-\omega) + E_2(\omega)c_{in}(\omega) + F_2(\omega)c_{in}^+(-\omega) \\ &+ E_2(\omega)d_{in}(\omega) + F_2(\omega)d_{in}^+(-\omega) + V(\omega)\xi(\omega), \end{aligned} \quad (7)$$

in which,

$$\begin{aligned} E_1(\omega) &= \frac{-i\hbar\sqrt{2\kappa_1}}{d(\omega)} (|a_s|^2 g_1^2 A_2 B_2 \\ &+ 2|c_s|^2 g_2^2 A_1 \Delta_2 + mN A_1 A_2 B_2), \\ F_1(\omega) &= \frac{-i\hbar\sqrt{2\kappa_1}|a_s|^2 g_1^2 A_2 B_2}{d(\omega)}, \\ E_2(\omega) &= \frac{-i\hbar\sqrt{2\kappa_2}a_s c_s g_1 g_2 A_1 A_2}{d(\omega)}, \\ F_2(\omega) &= \frac{i\hbar\sqrt{2\kappa_2}a_s c_s g_1 g_2 A_1 B_2}{d(\omega)}, \\ V(\omega) &= \frac{a_s g_1 A_1 A_2 B_2}{d(\omega)}, \end{aligned} \quad (8)$$

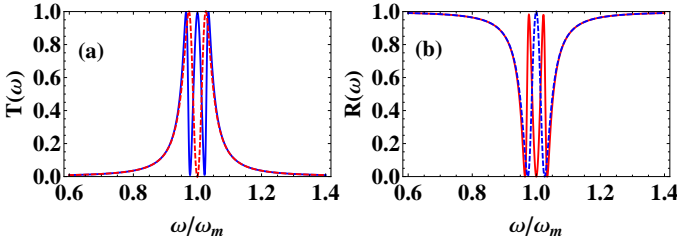


FIG. 2: (Color online) (a) The transmission spectrum $T(\omega)$ and (b) the reflection spectrum $R(\omega)$ of the single photon as a function of normalized frequency ω/ω_m when microwave pump field is turned off (red dashed line) and turned on (blue solid line). $\omega_p = 2\pi \times 7.1\text{GHz}$, $m = 48\text{ ng}$, $\omega_m = 2\pi \times 10.56\text{MHz}$, $\gamma_m = 2\pi \times 32\text{Hz}$, $\kappa_1 = 4\pi \times 100\text{KHz}$, $\kappa_2 = 2\pi \times 1\text{KHz}$, $\wp_l = 2 \times 65\text{ }\mu\text{W}$, $\wp_p = 6 \times 50\text{ nW}$ [24, 31, 32].

with

$$\begin{aligned} d(\omega) &= 2\hbar|a_s|^2 g_1^2 \Delta_1 A_2 B_2 + 2\hbar|c_s|^2 g_2^2 \Delta_2 A_1 B_1 \\ &\quad + m N A_1 B_1 A_2 B_2, \\ A_1 &= \Delta_1 + \omega + 2i\kappa_1, \quad B_1 = \Delta_1 - \omega - 2i\kappa_1, \\ A_2 &= \Delta_2 + \omega + 2i\kappa_2, \quad B_2 = \Delta_2 - \omega - 2i\kappa_2, \\ N &= \omega^2 + i\omega\gamma_m - \omega_m^2. \end{aligned} \quad (9)$$

Defining the spectrum of the field via $\langle a^+(-\Omega)a(\omega) \rangle = 2\pi S_a(\omega)\delta(\omega+\Omega)$, $\langle a(\omega)a^+(-\Omega) \rangle = 2\pi[S_a(\omega)+1]\delta(\omega+\Omega)$. The incoming vacuum field b_{in} and d_{in} are characterized by $\langle \tau(\omega)\tau^+(-\Omega) \rangle = 2\pi\delta(\omega+\Omega)$ ($\tau = b, d$) with $S_{bin} = S_{din}(\omega) = 0$. From Eq. (3) and Eq. (7), we find that the spectrum of the output fields has the form,

$$\begin{aligned} S_{aout}(\omega) &= R(\omega)S_{ain} + S^{(T)}(\omega) + S^{(V)}(\omega), \\ S_{bout}(\omega) &= T(\omega)S_{ain} + S^{(T)}(\omega) + S^{(V)}(\omega), \end{aligned} \quad (10)$$

where

$$\begin{aligned} R(\omega) &= |\sqrt{2\kappa_1}E_1(\omega) - 1|^2, \quad T(\omega) = |\sqrt{2\kappa_1}E_1(\omega)|^2, \\ S^{(T)}(\omega) &= 2\kappa_1|V(\omega)|^2\hbar\gamma_m m(-\omega)[1 + \coth(\frac{-\hbar\omega}{2\kappa_B T})], \\ S^{(V)}(\omega) &= 4\kappa_1|F_1(\omega)|^2. \end{aligned} \quad (11)$$

Single-photons multi-channel quantum router between microwave and optical light. — In Eq.(10), $R(\omega)$ and $T(\omega)$ are the contributions arising from the presence of a single photon in the input field. $S^{(v)}(\omega)$ is the contribution from incoming vacuum field. The $S^{(T)}(\omega)$ is the contributions from the fluctuation of the NR, respectively. Eq.(10) shows that even if there were no incoming photon, the output signals is generated via quantum and thermal noises. For the purpose of achieving a single-photons multi-channel quantum router, the key quantities are $R(\omega)$ and $T(\omega)$. Further, we also demonstrate the performance of the single-photon quantum router should not be deteriorated by the quantum and thermal noises terms $S^{(V)}(\omega)$ and $S^{(T)}(\omega)$.

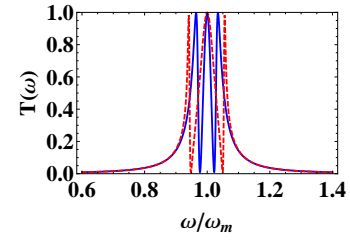


FIG. 3: (Color online) The transmission spectrum $T_1(\omega)$ of the single photon as a function of normalized frequency ω/ω_m with different microwave pump field power: $\wp_p = 6 \times 50\text{ nW}$ (blue solid line) and $\wp_p = 30 \times 50\text{ nW}$ (red dashed line). Other parameters take the same values as in Fig. 2.

To demonstrate the routing functions of the hybrid opto-electromechanics system, we first investigate the reflection $R(\omega)$ and transmission spectrum $T(\omega)$. For illustration of the numerical results, we choose the realistically reasonable parameters from the recent experiment [24, 31, 32]. $\omega_p = 2\pi \times 7.1\text{GHz}$, $m = 48\text{ ng}$, $\omega_m = 2\pi \times 10.56\text{MHz}$, $\gamma_m = 2\pi \times 32\text{Hz}$, $\kappa_1 = 2\pi \times 100\text{KHz}$, $\kappa_2 = 2\pi \times 1\text{KHz}$, $\wp_l = 2 \times 65\text{ }\mu\text{W}$, $\wp_p = 6 \times 50\text{ nW}$. We also apply the following conditions [23, 34], (i) $\Delta_1 = \Delta_1 \simeq \omega_m$ and (ii) $\omega_m \gg \kappa_1$. The first condition means that the optical cavity is driven by a red-detuned laser field which is on resonance with the optomechanical anti-Stokes sideband. The second condition is the well-known resolved sideband condition, which ensures the normal mode splitting to be distinguished [23].

The resulting spectra are shown in Fig 2. In the absence of the microwave pump field (*i.e.* $\wp_p = 0$). One can observe an inverted EIT and a standard EIT in the reflection and transmission spectra of a single photon. Note the $R(\omega_m) \approx 1$ and $T(\omega) \approx 0$. So the single photon is completely reflected at frequency ω_m . However, in the presence of the microwave pump field, the situation is completely different $R(\omega_m) \approx 0$ and $T(\omega_m) \approx 1$. More important, the reflection and transmission spectra of a single photon exhibit other two inverted dips and two normal dips at $\omega = \omega_m + \omega_0$ and $\omega = \omega_m - \omega_0$, here ω_0 is the small deviation from the central frequency ω_m which depend on the microwave pump field power. We can find $R(\omega_m \pm \omega_0) \approx 1$ and $T(\omega_m \pm \omega_0) \approx 0$. That is to say, the single photon is completely transmitted at frequency $\omega = \omega_m$, meanwhile, the single photons is completely reflected to the other two output ports at different frequencies $\omega_m + \omega_0$ and $\omega_m - \omega_0$. The physical effect can be explained by optomechanically induced transparency (OMIT) [23, 35] which originates from the radiation pressure coupling an optical mode to a mechanical mode. The OMIT depends on quantum interference and it is sensitive to phase disturbances. The coupling between microwave and the common NR breaks down the symmetry of the OMIT interference, then the single OMIT window is split into two transparency windows [36, 37].

Now, we can describe the working process of the multi-output quantum router between microwave and optical.

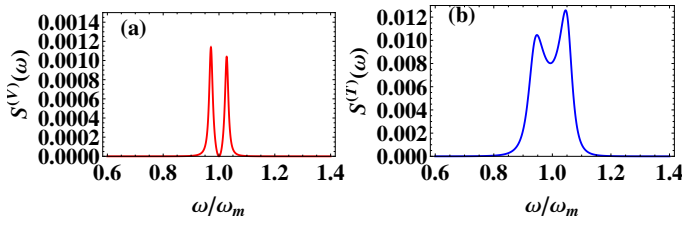


FIG. 4: (a) The vacuum noise spectrum $S^{(V)}(\omega)$ as a function of normalized frequency ω/ω_m . (b) The thermal noise spectrum $S^{(T)}(\omega)$ as a function of normalized frequency ω/ω_m . $T = 20\text{mK}$, other parameters take the same values as in Fig. 2.

When we turn off the microwave pump field, the single photons is complete reflected at frequency $\omega = \omega_m$ (*i.e.* $R(\omega_m) \approx 1$, $T(\omega_m) \approx 0$). However, when we turn on the microwave pump field, the single photons is complete transmitted at frequency $\omega = \omega_m$ (*i.e.* $R(\omega_m) \approx 0$, $T(\omega_m) \approx 1$), at the same time, three are completely reflected to the other two outputs at different frequencies $\omega = \omega_m + \omega_0$ and $\omega = \omega_m - \omega_0$ (*i.e.* $R(\omega_m \pm \omega_0) \approx 1$ and $T(\omega_m \pm \omega_0) \approx 0$). Fig. 3 describe the transmission spectrum $T(\omega)$ with the different microwave pump field power. From Fig. 3, we find the different output frequencies can be selected by adjusting the microwave pump field power, then it can be controlled the quantum signals to different IP address in quantum networks. Similarly, we can use the same principle to route the microwave signals by tuning optical light power, here, we do not describe it one by one.

Moreover, we discuss the effects of the quantum and thermal noise on the reflection and transmission spectrum of a single-photon. From Fig.4, the contribution of the vacuum noise maximum is about 0.13% and is thus insignificant. The thermal noise could be more critical in deteriorating the performance of the single-photon router. Clearly to beat the effects of thermal noise, the

number of photons in the probe pulse has to be much bigger than the thermal noise photons. However, if we work with NR temperatures like 20 mK, then the thermal noise term is insignificant as shown in Fig. 4(b).

In conclusion.— In this letter, we have proposed an experimentally accessible a bidirectional and tunable single-photons multi-channel quantum router between microwave and optical light based on the hybrid opto-electromechanical system. The system consist of a microwave circuit and an optical toroidal nanocavity are coupled to a common NR. It is the first demonstration that the single-photon of optical frequency can be routed into three different output ports by adjusting microwave power. More important, the two output optical signals can be selected according to microwave power, then it can be controlled the quantum signals to different IP address in quantum networks. Meanwhile, we also demonstrate the vacuum and thermal noise will be insignificant for the optical performance of the single-photon router at temperature of the order of 20 mK. Our proposal may have paved a new avenue towards multi-channel router and quantum network.

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